MITSUBISHI ELECTRIC RESEARCH LABORATORIES
Cambridge, Massachusetts

On Closure Relations for Dynamic Vapor Compression Cycle Models

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Model-Based System Design

Model-based heat pump design

- Faster, cheaper development
- Systematically design multivariable dynamics

Are emergent dynamics from composed models consistent with experimental data?

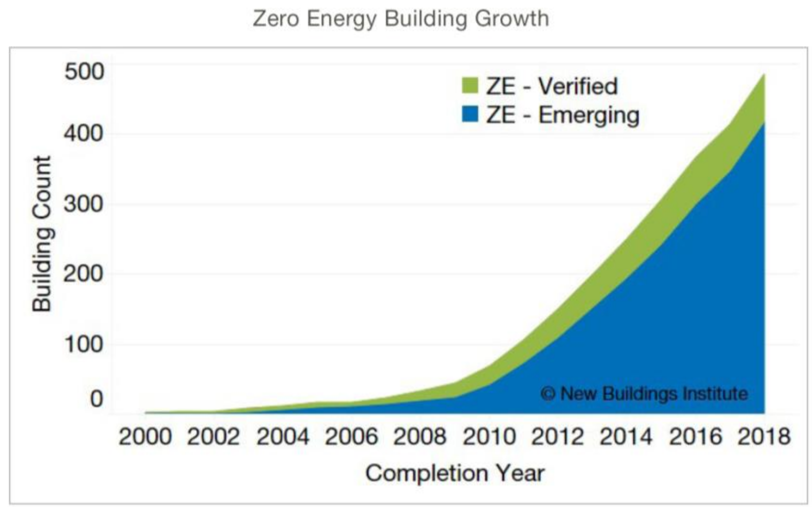
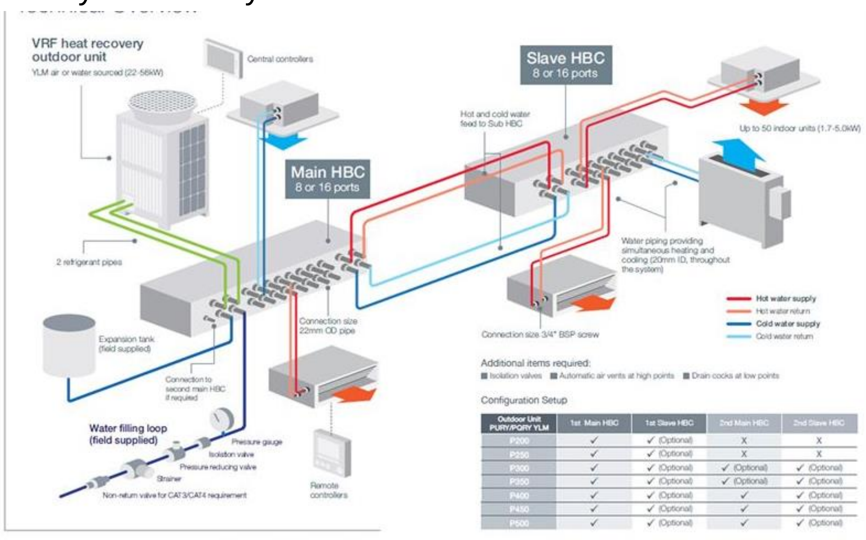


Fig 1. The Buildings List includes nearly 500 projects and is on a steep curve upward, having increased over 700% since 2012.

Room air-conditioner

Hybrid VRF system



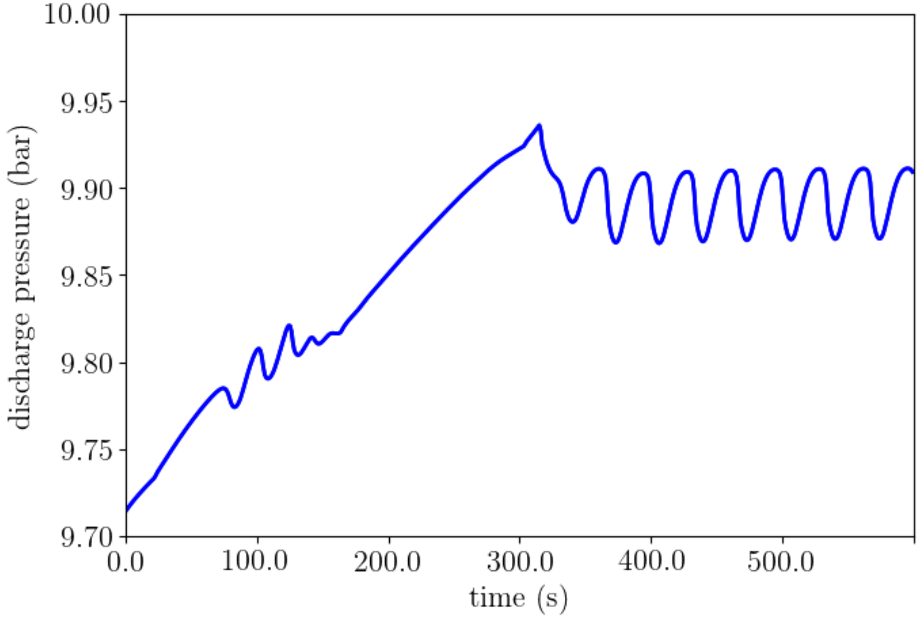
The Challenge of Closure Relations

- HTC or frictional dP
- Not formulated for computational efficiency – can be problematic
- What are the tradeoffs of different approaches?

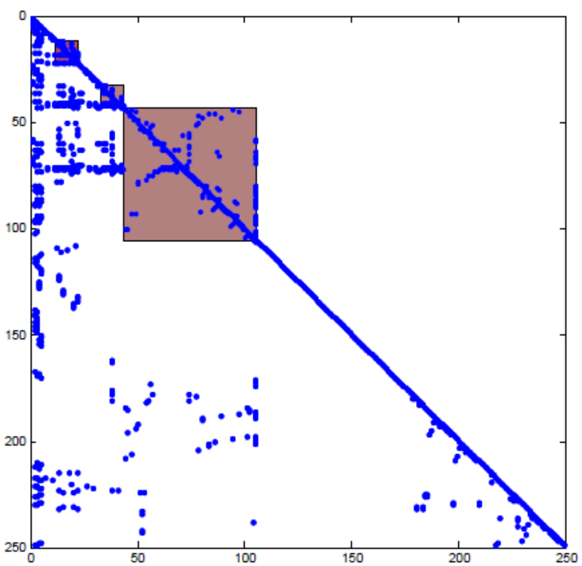
Sieder-Tate heat transfer correlation

$$\alpha D/k = 0.027 Re_D^{4/5} Pr^{1/3} \left(\frac{\mu}{\mu_{surf}}\right)^{0.14}$$

Nonphysical pressure variations

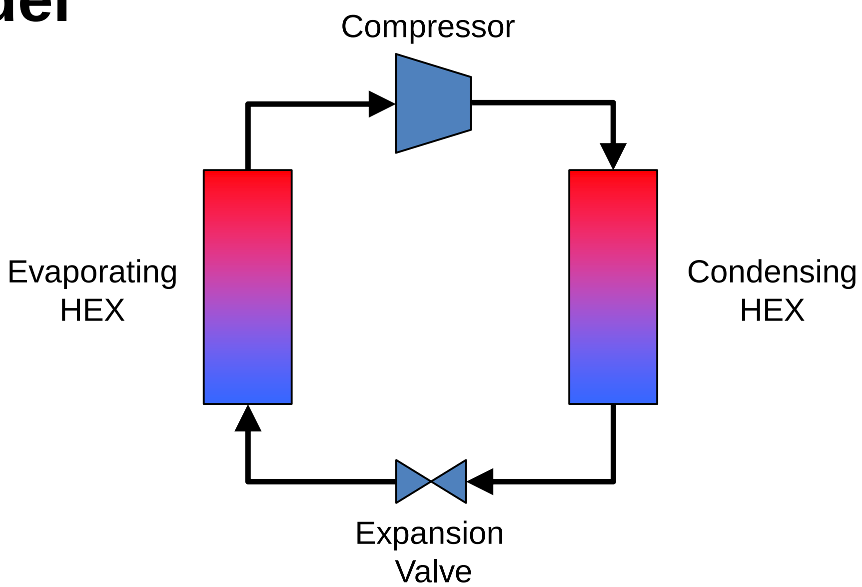


BLT system Jacobian



Vapor Compression Cycle Model

- System dynamics are dominated by heat exchangers
- Refrigerant properties defined via technical equation of state
  - Pressure and specific enthalpy
- Compressor and valve performance curves defined by the user



Dynamic HEX model

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho A v)}{\partial x} = 0$$

$$\frac{\partial(\rho v A)}{\partial t} + \frac{\partial(\rho v^2 A)}{\partial x} = -A\frac{\partial p}{\partial x} - F_f$$

$$\frac{\partial(\rho u A)}{\partial t} + \frac{\partial(\rho v h A)}{\partial x} = vA\frac{\partial p}{\partial x} + vF_f + \frac{\partial Q}{\partial x}$$

Static compressor model

$$\eta_v(f, P_r, \dots) = \frac{\dot{m}_{comp}}{\rho_{suc} V f}$$

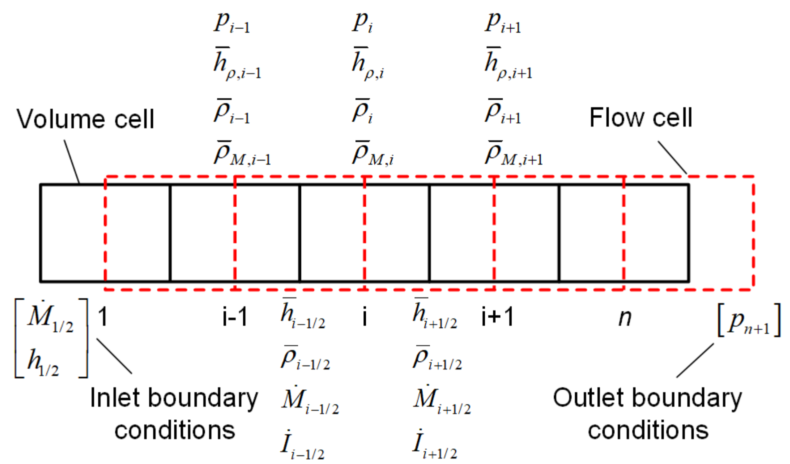
$$\eta_{is}(f, \dots) = \frac{h_{dis,isen} - h_{suc}}{h_{dis} - h_{suc}}$$

Static LEV model

$$\dot{m} = C_v a_v \sqrt{\rho_{in} \Delta P}$$

Heat Exchanger Models

- Finite volume dynamic HEX model
  - -16 volumes / HEX
- 1-D pipe wall model with dehumidification
- Ideal gas mixture for moist air



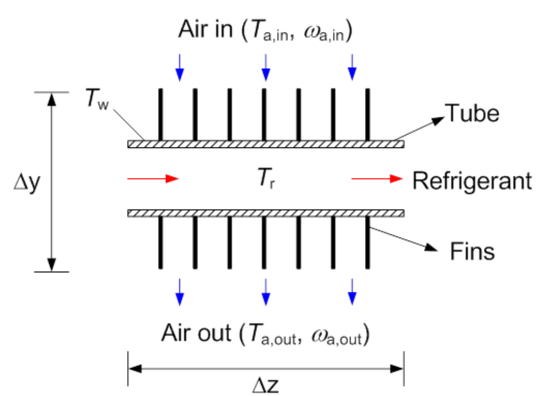
Refrigerant-side finite volume discretization

Conservation equations

$$A_c \Delta z \rho_i = \dot{M}_{i-1/2} - \dot{M}_{i+1/2}$$

$$\Delta z \frac{\mathrm{d}\dot{M}_{i+1/2}}{\mathrm{d}t} = \dot{I}_i - \dot{I}_{i+1} - A_c(p_{i+1} - p_i) - P\Delta z \bar{\tau}_{w,i+1/2},$$

$$A_c \Delta z u_i = \dot{M}_{i-1/2}(h_{i-1/2} - \bar{h}_{\rho,i}) - \dot{M}_{i+1/2}(h_{i+1/2} - \bar{h}_{\rho,i}) + P\Delta z q_i'',$$



Air-side finite volume discretization

Momentum Balance Approximations

Original form:
3n dynamic states

$$\Delta z \frac{\mathrm{d}\dot{M}_{i+1/2}}{\mathrm{d}t} = \dot{I}_i - \dot{I}_{i+1} - A_c(p_{i+1} - p_i) - P\Delta z \bar{\tau}_{w,i+1/2},$$

Friction only:
2n dynamic states

$$p_{i+1} = p_i - \frac{P}{A_c}\Delta z \bar{\tau}_{w,i+1/2}$$

Uniform dp/dt:
2n+1 dynamic states

$$\frac{dp_1}{dt} = \frac{dp_{n+1}}{dt} = \frac{dp_{ref}}{dt}$$

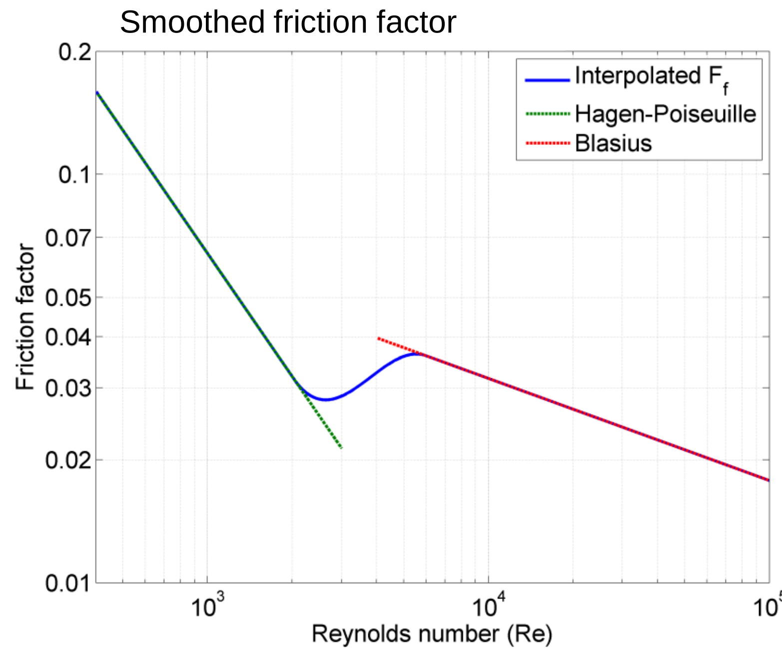
Linear Δp:
n+3 dynamic states

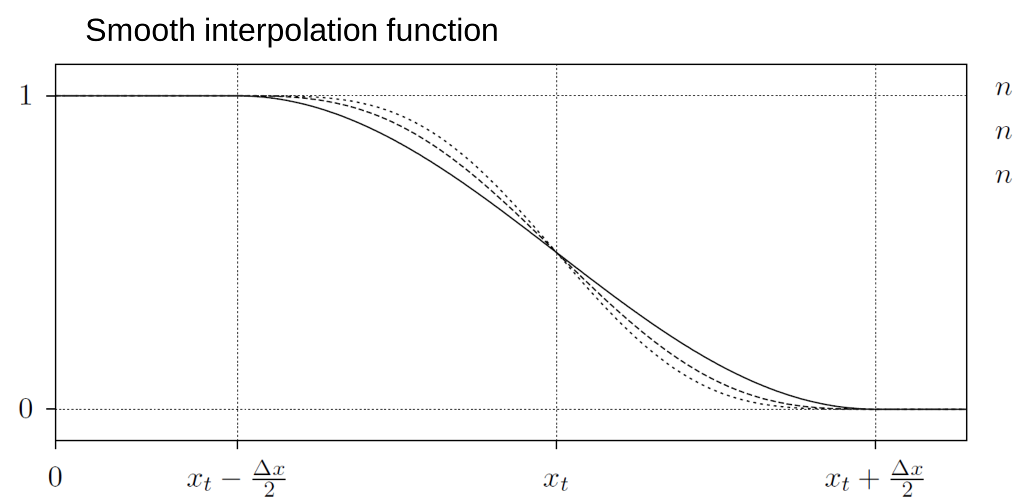
$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = \left(1 - \frac{i-1}{n}\right)\frac{\mathrm{d}p_1}{\mathrm{d}t} + \frac{i-1}{n}\frac{\mathrm{d}p_{n+1}}{\mathrm{d}t}$$

$$n\Delta z \frac{\mathrm{d}\bar{\dot{M}}}{\mathrm{d}t} = \dot{I}_1 - \dot{I}_{n+1} - A_c(p_{n+1} - p_1) - P\Delta z \sum_{i=1}^{n} \bar{\tau}_{w,i+1/2}$$

Closure Relations

- Can be implemented as model objects
- Typically defined over distinct flow regimes
  - Laminar / turbulent
  - Condensing / evaporating
- Hard to predict flow regime after DAE initialization
  - Discontinuities can result in chattering or crashing
- Smooth interpolation can help
  - But not necessarily with nonlinear algebraic loops





$$z(x) = T(x) \cdot f(x) + \big(1 - T(x)\big) \cdot g(x)$$

Heat Transfer Coefficients: Algebraic

- Multiple regimes can be blended together to form a universal function

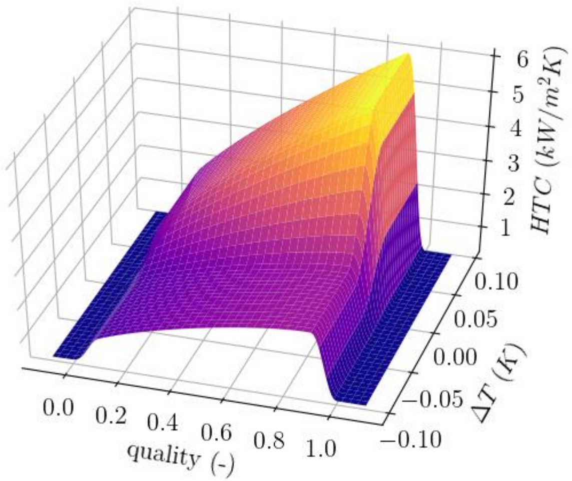
- Simplified closure relations can approximate full correlations, e.g.,

$$\alpha = \alpha_0 \left(\frac{\dot{m}}{m_0} \right)^b$$

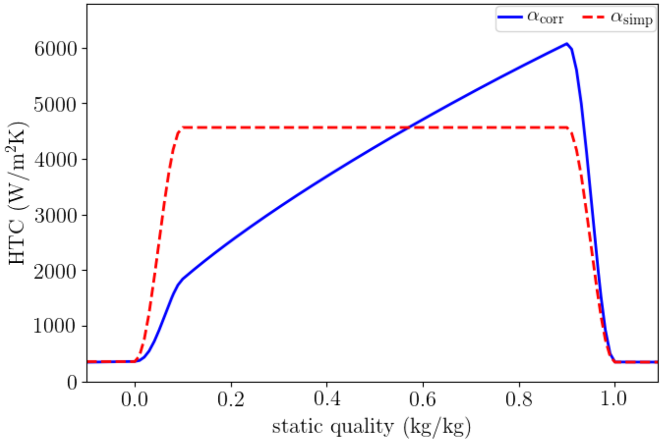
- Reduce size of nonlinear blocks
  - Improved robustness
  - Faster simulation
  - Invertible

- May be physically justified

Full blended HTC



Simplified condensing HTC



Heat Transfer Coefficients: Dynamic

- α and Δp often do not change instantaneously
  - Unmodeled low-pass dynamics, e.g., refrigerant / oil behavior

- Explicitly adding additional dynamics can improve model behavior
  - Decouple nonlinear equation blocks, improves simulation speed
  - Easy in equation-oriented languages

- Trading one set of errors for another (more manageable) set of errors

```
alphas[k] = simpleHTC_FullRegion(xs[k], alpha_vap,
                    alpha_2ph, alpha_liq, xdot_1, xdot_2);
```

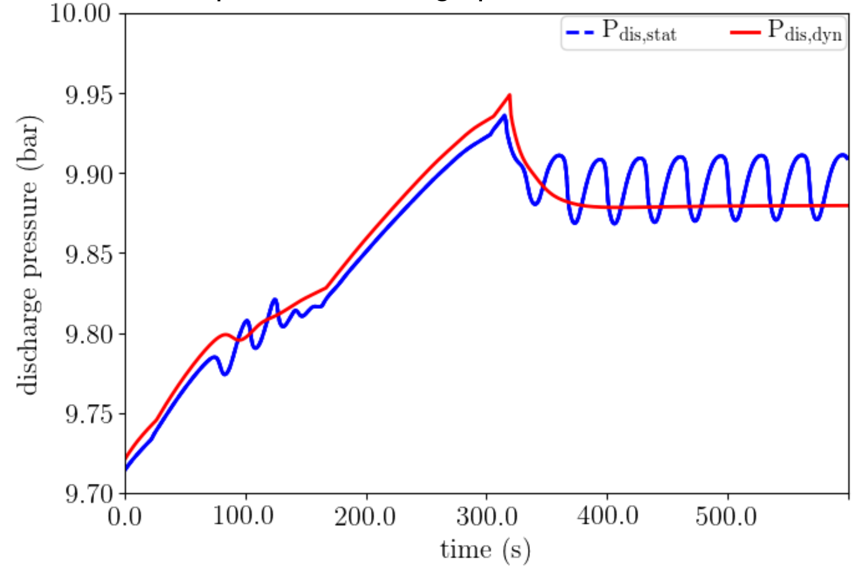
↓

```
alphaHats[k] = simpleHTC_FullRegion(xs[k], alpha_vap,
                    alpha_2ph, alpha_liq, xdot_1, xdot_2);
der(alphas[k]) = (alphaHats[k] – alphas[k])/tau;
```

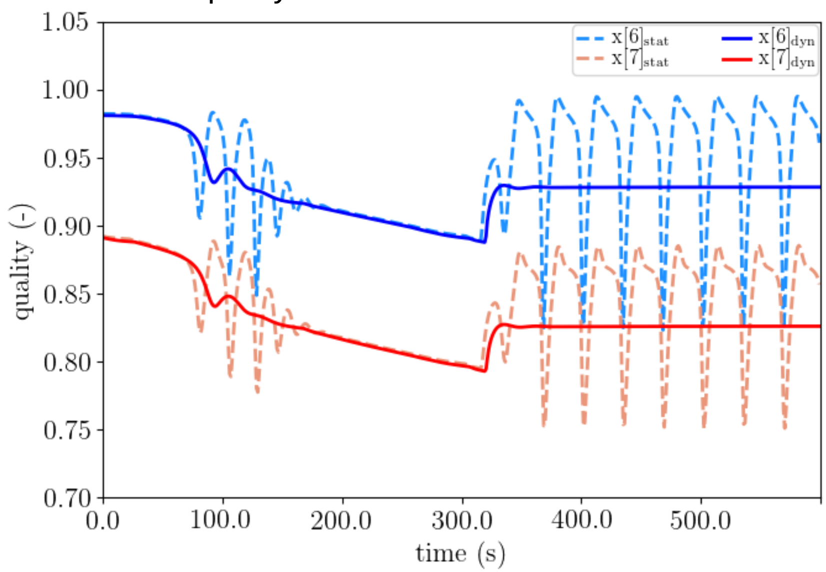
Results

- Simplified algebraic vs. dynamic correlation in cycle
- Significantly reduced oscillations
  - Accurately captures low-frequency behavior
- Improved system linearization

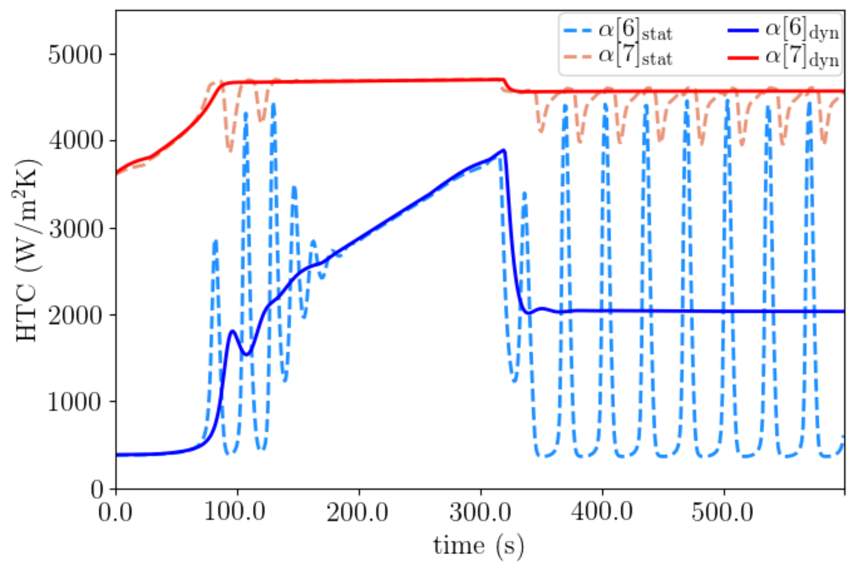
Compressor discharge pressure



Flow quality



Condensing heat transfer coefficient



Computational Performance

- Dynamic models are much faster
  - Less nonlinear equation blocks
  - Less oscillations
- Enables use of correlations from literature in system models
- Dynamic models do not help as much when frictional dp relations can be inverted
- Sensitive to choice of time constant

HTC Formulation	CPU time (s)
Algebraic, correlation	-
Algebraic, simplified	228
Dynamic, correlation	146
Dynamic, simplified	39

dp Formulation	Algebraic CPU time (s)	Dynamic CPU time (s)
Standard	2855	-
Friction only	111	-
dp/dt	-	214
Linear pressure	-	175

Summary

- The formulation of closure relations can affect the behavior and numerical performance of a simulation model
- Simpler algebraic closure relations may be justified by physics-oriented reasoning for some applications
- Dynamic closure relations may also be justified
  – “Accurate” model produces non-physical behavior, “approximate” model produces more realistic behavior
- Choice of type of closure relations must be made in light of numerical and behavioral considerations
  – Potentially significant effects on simulation performance and speed

Thank you for your time!

Any questions?

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